

WAVE PROPAGATION IN INHOMOGENEOUS ANISOTROPIC RECTANGULAR WAVEGUIDES  
BY THE EFFECTIVE INDEX METHOD

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ABSTRACT

Hybrid mode dispersion and mapping of multilayer rectangular diffused birifrangent waveguides are studied by the effective index method for two orientations, horizontal and vertical, of the crystal optic axis. At first the structure is examined in the approximate lossless approach, then the perturbation technique allows us to evaluate the extinction coefficient value to employ as starting point in the direct search optimization strategy for determining the complex propagation constant for the exact solution. The guided  $E_{11}^x$  and  $E_{11}^y$  modes exhibit almost the same cutoff wavelength and their dispersion curves little differ from the ones of corresponding  $TM_0$ , up to  $1 \mu m$  for vertical optic axis because of the surface plasma waves, and  $TE_0$  modes of the slab waveguide without and with the metal film respectively.

Introduction

Anisotropic indiffused waveguides are very interesting in the realization of multilayer structures for the fabrication of integrated optics devices and systems. A multilayer guide, including a thin film overlay and an embedded metal film, presents the advantage of a fine tuning of the propagation constant, by varying the overlay thickness or the refractive index<sup>1</sup> and a reduction of the ohmic losses<sup>2</sup>. Therefore, by these guiding structures, it is possible to realize complex and multifunctional devices.

Several authors<sup>3,4,5</sup> investigated the mode properties of slab multilayered dielectric waveguides but, till now, the optical anisotropic rectangular guides have not been examined in the proper way.

In this paper we determine the dispersion relations and the field distribution of anisotropic optical guides (see Fig. 1) whose rectangular cross section consists of a Ti diffused  $LiNbO_3$  ferroelectric substrate overlayed by an appropriate oxide buffer having a re-

fractive index  $n_{ov}$  and a thickness  $b$  with two embedded Ag electrodes modeled by an  $\epsilon_m$  complex dielectric constant and thickness  $h$ . The structure is investigated in the approximate lossless formulation and, then, the exact lossy solution is obtained by a direct search optimization strategy where the extinction coefficient  $k$  starting values are estimated by the perturbation technique.

Moreover in order to assess the influence of the optic axis orientation on the electromagnetic field and, therefore, the best configuration for the design of integrated optics devices, we investigate the waveguide both for c horizontal ( $\parallel$ ) and vertical ( $\perp$ ) under the hypothesis of a diagonal permittivity tensor.

This waveguiding system supports<sup>6,7</sup> the  $E_{pq}^x$  and  $E_{pq}^y$  hybrid modes that are TE or TM in respect to one transverse coordinate. More precisely, for uniaxial materials with optic axis horizontal or vertical, the guided waves are derivable from a c-directed hertzian potential. Since the rigorous solution of the two-dimensional wave equation is considerably complicated even for a not diffused and isotropic guide, the geometry analysis is carried-out by the effective index method<sup>9</sup> that is easy to use and gives a noticeable precision together with a short C.P.U. computer time after determining the slab guide characteristics by the transformation matrix method<sup>5,10,11</sup>.

Among the other results, it is worth while to notice that, for a silver film  $0.02 \mu m$  thick, the guide configuration ( $\parallel$ ) does not support surface plasma waves that are excited in the configuration ( $\perp$ ) at the wavelengths less than  $1 \mu m$ .

Mode analysis by the effective index method

The multi-dielectric structure is depicted in Fig.1. The lithium niobate perovskite crystal is Ti-diffused according to the complementary error function in the x direction with maximum index changes  $\Delta n_e$  and  $\Delta n_o$  of ex-

FIGURE 1: CHANNEL MULTILAYER WAVEGUIDE FOR ELECTROOPTICAL MODULATOR REALIZATION.

extraordinary  $n_e(x)$  and ordinary  $n_o(x)$  indices in respect to the bulk substrate  $n_{es}$ ,  $n_{os}$  ones:  $n_{e,o}(x)=n_{es,os} + \Delta n_{e,o} \operatorname{erfc}|x/d|$ , ( $d$ =diffusion depth).

For uniaxial crystals if the optic axis is horizontal, the  $E_{pq}^x$  and  $E_{pq}^y$  modes supported by the optical guide are respectively transverse electric or magnetic to  $y$  coordinate while for vertical  $c$ -axis the  $z$  traveling hybrid modes are  $TE_x$  or  $TM_x$ .

In fact, in the last case, the  $E_{pq}^x$  mode are obtained by putting  $H_x=0$  and solving the Maxwell's equations in respect to the  $H_y$  component that, in the diffused core, must satisfy the equation:

$$\frac{n_e^2}{n_o^2} \frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + n_e^2 \frac{\partial}{\partial x} \left( \frac{1}{n_e^2} \right) \frac{\partial H_y}{\partial x} + k_o^2 (n_e^2 - \bar{n}_{eq}^2) H_y = 0 \quad (1)$$

where it was supposed a time harmonic variation and a normal mode one  $\exp(-jk_{oneq}z)$  along the propagation direction;  $\bar{n}_{eq} = n - jk$  is the complex equivalent index and  $k_o = 2\pi/\lambda_o$  is the free space wavenumber.

Let  $H_y(x,y) = \varphi(x) \psi(y)$  be separable the equation (1) becomes

$$\frac{1}{\varphi} \frac{n_e^2 d^2 \varphi}{n_o^2 dx^2} + n_e^2 \frac{d}{dx} \left( \frac{1}{n_e^2} \right) \frac{1}{\varphi} \frac{d \varphi}{dx} + k_o^2 (n_e^2 - \bar{n}_{eq}^2) = -\frac{1}{\psi} \frac{d^2 \psi}{dy^2} \quad (2)$$

The left side with  $\partial/\partial y=0$ , by imposing the boundary conditions at the interfaces  $x=b$ ;  $x=h$ ;  $x=0$  and  $x=-t$ , gives rise to the  $TM_z$  modes whose equivalent indices  $\bar{n}_{eq,s}$  depend on both  $n_o$  and  $n_e$  but chiefly on  $n_e$  because the real part  $n_s$  of  $\bar{n}_{eq,s}$  ranges between  $n_{es}$  and  $(n_{es} + \Delta n_e)$ . Afterwards the result

$$\frac{1}{\varphi} \frac{n_e^2 d^2 \varphi}{n_o^2 dx^2} + n_e^2 \frac{d}{dx} \left( \frac{1}{n_e^2} \right) \frac{1}{\varphi} \frac{d \varphi}{dx} = -k_o^2 (n_e^2 - \bar{n}_{eq}^2) \quad (3)$$

is back substituted into (2) to get

$$\frac{d^2 \psi}{dy^2} + k_o^2 (\bar{n}_{eq,s}^2 - \bar{n}_{eq}^2) \psi = 0 \quad (4)$$

that is the wave equation for the trapped  $TE_z$  modes when  $\partial/\partial x=0$ ,  $\bar{n}_{eq}$  being the desired index of the original rectangular structure obtained by imposing the continuity conditions of the interfaces  $y=g$  and  $y=(w+g)$ .

Likewise, to obtain the  $E_{pq}^y$  waves we must put in the Maxwell's equation  $E_x=0$  and couple the  $\bar{n}_{eq,s}$  relative to the  $TE_z$  modes of the component ( $\partial/\partial y=0$ ) planar wave guides to the  $TM_z$  polarization with  $\partial/\partial x=0$ .

The equation (3) relative to the three different guides indefinite along the  $y$  axis of Fig. 1 was solved as in 5,10 by the method of the transformation matrix relating the two continuous field components at the interfaces  $x=0$  and  $x=-t$  where the diffusion process may be thought exhausted. The equation (3) splits in a system of two first-order differential equations that is integrated by employing the Gear's<sup>12</sup> predictor-corrector technique that allows to reach the required numerical precision for arbitrary index profile also for wide integration range with a not excessive C.P.U. computer time. The zeros of the real eigenvalue equation are found by the secant method while the minima in the lossy approach are reached by the razor technique<sup>13</sup>.

## Results and discussion

In Fig. 2 are reported the lossy  $TM_z$  and  $TE_z$  normalized equivalent indices

$$N = \frac{n_{TM,TE}^2 - n_{es,os}^2}{(n_{es,os} + \Delta n_{e,o}) - n_{es,os}^2} \quad (5)$$

vs.  $k_o$  for the ( $\perp$ ) configuration with  $n_{ov}=2.10$ ;  $b=0.4\mu\text{m}$ ;

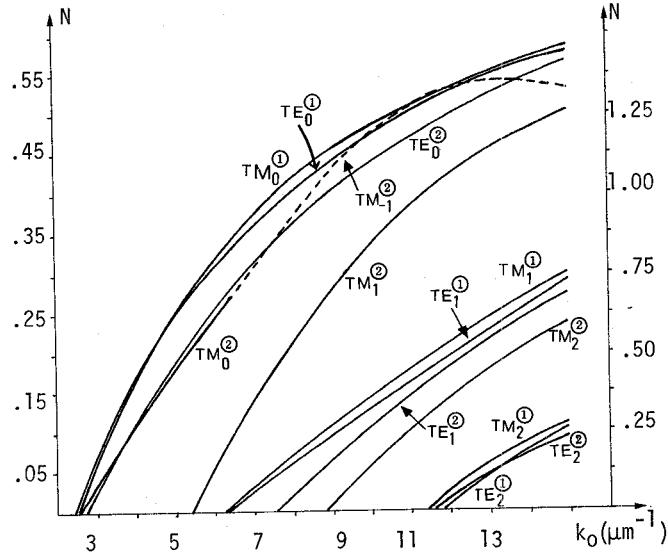


FIGURE 2: TE AND TM DISPERSION CURVES RELATIVE TO (1) AND (2) GUIDES FOR C VERTICAL.

$\epsilon_m = -16.3 - j0.522$ ;  $h = 0.02\mu\text{m}$ . The dispersion curves (1) superscript refer to the slab guide (1) without metal film and the ones with (2) to the guide (2) with silver layer. It is worth to notice that the  $TE_z(\parallel)$  equivalent index values of the slab (1) with horizontal optic axis differ from the  $TM_z(\perp)$  ones relative to the vertical  $c$  configuration only on the fifth significant figure. The same applies to the  $TM_z(\parallel)$  and  $TE_z(\perp)$ . This correspondence only in part occurs in the slab with me-

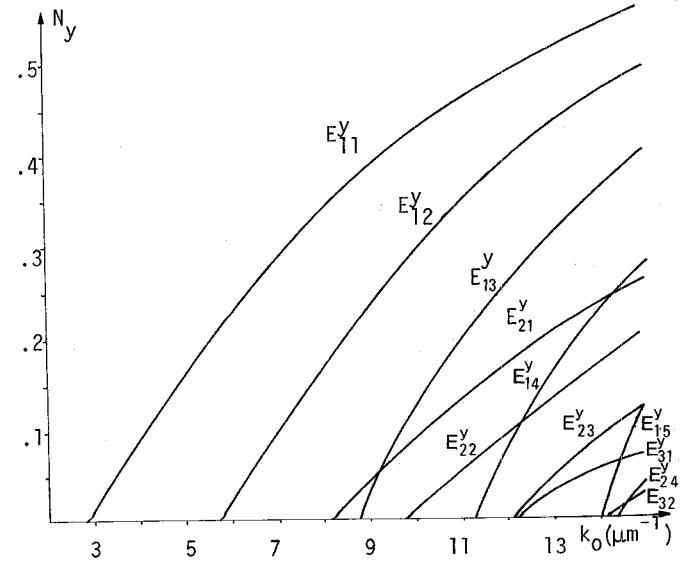


FIGURE 3:  $E_y^y$  DISPERSION CURVES FOR C HORIZONTAL CONFIGURATION. THE PARAMETERS ARE AS IN THE FIG.2.

tal layer because the differences concern the third significant figure. Moreover, the TM polarization behaviour is rather different for the two configurations since the (||) configuration does not support surface plasma waves (SPW) while for c vertical the  $TM_0$  wave suffers a transformation in  $TM_1$  for wavelengths less than  $1\mu m$  and tilts in its upper part because of the  $LiNbO_3$  intrinsic dispersion. The extinction coefficients of the TE modes are two magnitude orders lower than the TM ones and there are absorption peaks near to  $1\mu m$ .

In Fig. 3 are drawn the lossless  $E_{pq}^y$  indices  $N_y$  normalized by (5) to the  $n_{es}$  substrate index with the geometrical and physical parameters of Fig. 2 and  $g=w=0.75\mu m$  and relative to the configuration with horizontal optic axis.

The lossy  $n$  values differ from the lossless ones only on the tenth significant figure and no changes were found on the extinction coefficients in respect to the ones calculated via the perturbation approach.

Besides we point-out the fundamental mode without metal is the  $TE_0$  wave while the  $TM_0$  mode is the lower order field in the slab with metal. This result leads to cutoff wavelengths almost equal for the  $E_{11}^x$  and  $E_{11}^y$  guided modes since the real part of their refractive indices little differ from the  $TM_0$  fundamental mode of the slab without metal layer and the  $TE_0$  wave of the slab with metal layer respectively. At last we see that the  $E_{pq}^x$  and  $E_{pq}^y$  waves are alternatively even and odd modes in respect to the plane  $y=0$  that behaves as an electric or magnetic wall. In Fig. 4 are drawn the extinction coefficients of the  $E_{pq}^y$  modes. Also these curves show peak values at  $\lambda_0=1\mu m$ .

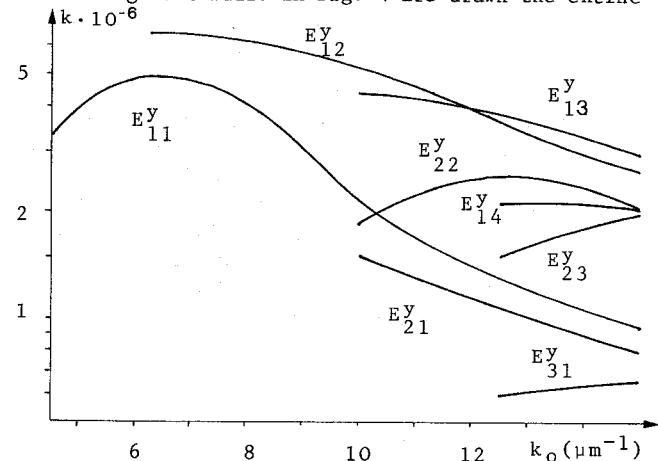


FIGURE 4:  $E_{pq}^y$  EXTINCTION COEFFICIENTS FOR C-HORIZONTAL.

tion coefficients of the  $E_{pq}^y$  modes. Also these curves show peak values at  $\lambda_0=1\mu m$ .

The Fig. 5 shows the lossy ( $\perp$ ) indices  $N_x$  of the  $E_{pq}^x$  modes as a function of  $k_0$ . The  $E_{pq}^x$  curves are considerably different for the two orientations of the optic axis. For the ( $\perp$ ) case we find the fundamental mode  $E_{11}^x$  only for  $\lambda_0 \leq 1\mu m$ . For  $\lambda_0=1\mu m$  we find also the  $E_{21}^x$  mode lacking in the (||) structure. Also in this case the differences between complex perturbed index values and the lossy exact ones are insignificant and the attenuations range between 1 dB/cm and the peak value of 169 dB/cm.

In this case the lossy fundamental  $TM_0$  indices are rather different from the  $E_{11}^x$  ones. Moreover, the lossy  $TM_z$   $n$  values for the slab ② differ on the third significant figure from the lossless ones.

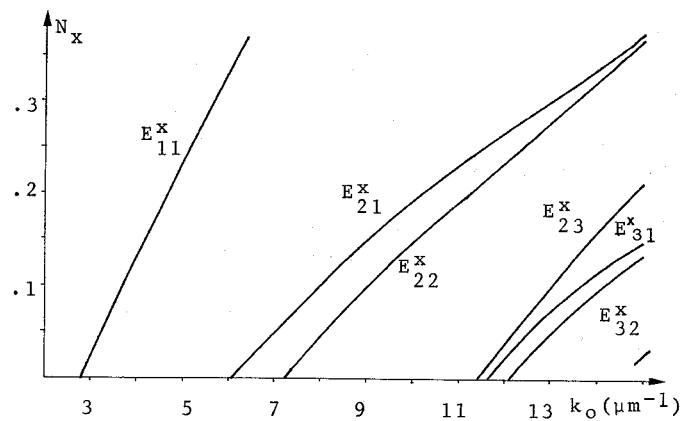


FIGURE 5:  $E_{pq}^x$  NORMALIZED INDICES vs.  $k_0$  FOR C-VERTICAL.

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